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Event-triggered adaptive tracking control for stochastic nonlinear systems under predetermined finite-time performance

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Summary

In this paper, an event-triggered adaptive tracking control strategy is proposed for strict-feedback stochastic nonlinear systems with predetermined finite-time performance. Firstly, a finite-time performance function (FTPF) is introduced to describe the predetermined tracking performance. With the help of the error transformation technique, the original constrained tracking error is transformed into an equivalent unconstrained variable. Then, the unknown nonlinear functions are approximated by using the multi-dimensional Taylor networks (MTNs) in the backstepping design process. Meanwhile, an event-triggered mechanism with a relative threshold is introduced to reduce the communication burden between actuators and controllers. Furthermore, the proposed control strategy can ensure that all signals of the closed-loop system are bounded in probability and the tracking error is within a predefined range in a finite time. In the end, the effectiveness of the proposed control strategy is verified by two simulation examples.

KEYWORDS

adaptive backstepping method, event-triggered control, multi-dimensional Taylor networks, predetermined finite-time performance, stochastic nonlinear systems

1 | INTRODUCTION

It is well known that many practical systems are often unstable due to the presence of stochastic perturbations. Therefore, the control design of stochastic nonlinear systems has become a hot research topic, and many control methods have been proposed, such as adaptive control,[1–3] sliding mode control,[4] and robust control.[5] Specifically, adaptive control has become an effective method to solve the control problems of stochastic nonlinear systems. Meanwhile, by combining the approximation-based intelligent control method with the traditional adaptive backstepping method, many meaningful research results have been achieved, such as neural network (NN) control,[6–9] fuzzy control,[10,11] and multi-dimensional Taylor network (MTN) control.[12,13] Specially, since MTN has the advantages of simple structure, small computational effort and fast function approximation,[14] MTN-based control method has gained more and more attention and been successfully applied to different types of stochastic nonlinear systems, such as stochastic nonlinear systems with input constraints,[15–17] stochastic nonlinear systems with multiple faults,[13] and large-scale stochastic nonlinear systems.[18] However, most of the above results focused on the control performance of the system while neglected the limitations of control resources. Recently, a growing attention has been paid on event-triggered control (ETC), which has important theoretical significance and practical application value.

In recent years, event-triggered control has been rapidly developed because of its advantages such as low economic cost, high flexibility, and good operability. In view of the limited network resources, it is crucial to reduce the utilization of communication resources while ensuring the control performance. For this reason, the authors in Reference 19 first proposed the event-triggered mechanism for first-order stochastic systems, which can reduce redundant transmissions by discrete signal transmission. Since then, ETC-based approach not only has achieved many valuable results in nonlinear systems,[20–22] but also has been successfully extended to stochastic nonlinear systems[23–27] and stochastic nonlinear multi-agent systems.[28] However, although ETC has achieved fruitful results, most of the above results focused only on achieving tracking effects without considering the predetermined finite-time performance of the controlled system.

In practical systems, the tracking error usually needs to meet performance specifications, such as fast convergence and small steady-state error, to ensure the control performance of the systems. Many scholars have exerted great effort in addressing this issue. To improve the convergence speed and robustness of the closed-loop system, the authors in References 29,30 proposed a novel practically predefined-time control scheme for stochastic nonlinear systems, which can achieve the tracking error converges to a small neighborhood of the origin in the predefined-time sense. To enhance the tracking performance of the system, prescribed performance control method, which means the tracking error must satisfy prescribed boundary conditions, has been widely applied to general nonlinear systems[31,32] and stochastic nonlinear systems.[33–36] Regrettably, the above prescribed performance control results did not consider the predetermined finite-time performance problem. As a matter of fact, the control objectives of many demanding practical systems, such as robotic systems[37,38] and flight systems,[39,40] are usually desired to be achieved in a finite time. In this context, the authors in Reference 41 proposed the event-triggered control protocol to achieve finite-time consensus for the second-order leader-following nonlinear multi-agent system. Moreover, the predetermined finite-time performance control has received extensive attention and led to many valuable results.[42–45] However, the problem of event-triggered adaptive tracking control for stochastic nonlinear systems with predetermined finite-time performance has received little attention, which motivates us to carry out this work.

Based on the above analysis, the control problem of achieving the predetermined finite-time tracking performance for strict-feedback stochastic nonlinear systems is considered in this paper. An event-triggered adaptive tracking control strategy with a relative threshold is proposed by using the adaptive backstepping method and MTN, which can guarantee that all signals of the closed-loop system are bounded in probability. Compared with the existing results, the novel contributions of this paper can be summarized as follows:

1. A unified adaptive control framework is proposed to design the event-triggered adaptive tracking controller for strict-feedback stochastic nonlinear systems with predetermined finite-time performance by integrating MTN-based approach, adaptive backstepping method and stochastic stability theory. The proposed control strategy ensures that all the closed-loop signals are bounded in probability and the tracking error converges to a predefined region in a finite time. Although many event-triggered control strategies have been developed in References 24,27,43, the above results cannot be directly used to solve predetermined finite-time performance control problems for stochastic nonlinear systems.
2. Although many meaningful MTN-based results have been proposed for stochastic nonlinear systems,[12,13,15] the issues of predetermined finite-time performance control and ETC were not considered simultaneously. Even though the predetermined finite-time performance control problem was studied in References 42,44,46, their control objects were nonlinear systems, which ignored the presence of stochastic disturbances. In addition, the authors in References 33,34 addressed the predetermined performance control problem for stochastic nonlinear systems, while they did not consider finite-time control or ETC. Therefore, the problem and the system studied in this paper are more generalized.
3. In order to overcome the difficulties in controller design brought by the constrained tracking error, a coordinate transformation function is introduced, which can transform the constrained tracking error into an equivalent unconstrained variable. Besides, an event-triggered mechanism with a relative threshold is introduced to reduce the communication burden between actuators and controllers. Moreover, the control strategy proposed has the advantages of simple structure, small computation and easy implementation with the aid of MTN.

2 | PRELIMINARY PREPARATION OF PROBLEMS

2.1 | Problem formulation

In this paper, consider a class of strict-feedback stochastic nonlinear systems as follows

$$\begin{cases} dx_i = \left(x_{i+1} + f_i\left(\overline{\boldsymbol{x}}_i\right)\right)dt + g_i^{\mathrm{T}}\left(\overline{\boldsymbol{x}}_i\right)d\omega, i = 1, \dots, n-1 \\ dx_n = \left(u + f_n\left(\overline{\boldsymbol{x}}_n\right)\right)dt + g_n^{\mathrm{T}}\left(\overline{\boldsymbol{x}}_n\right)d\omega \\ y = x_1 \end{cases} \tag{1}$$

where $x_1, x_2, \dots, x_n$ denote the system states and $\overline{\boldsymbol{x}}_i = [x_1, \dots, x_i]^{\mathrm{T}} \in R^i$, for $i = 1, 2, \dots, n$. u and y represent the control input and measurement output of the system, respectively. ω stands for an independent r-dimensional Wiener process. $f_i\left(\overline{\boldsymbol{x}}_i\right) : R^i \to R$ and $g_i\left(\overline{\boldsymbol{x}}_i\right) : R^i \to R^r$, $i = 1, 2, \dots, n$ represent unknown smooth nonlinear functions satisfying $f_i(\mathbf{0}) = 0$ and $g_i(\mathbf{0}) = \mathbf{0}$.

For system (1), the work of this paper is to propose an event-triggered adaptive tracking control strategy, which can achieve the following objectives

1. all the closed-loop signals are bounded on $[0, +\infty)$ in probability.
2. the tracking error $e_1 = y - y_d$ converges to a predefined region in a finite time.

The following Assumption and Lemmas are required for controller design.

Assumption 1 (47). The reference trajectory y_d and its time derivatives up to the n-th order are continuous and bounded.

Lemma 1 (1, Young's inequality). *For any given* $(x, y) \in R^2$, *the following inequality holds*

$$xy \le \frac{\beta^p}{p}|x|^p + \frac{1}{q\beta^q}|y|^q, \tag{2}$$

where $\beta > 0$, $p > 1$, $q > 1$, and $(p-1)(q-1) = 1$.

Lemma 2 (43). *For* $\forall \xi > 0$ *and* $\forall \tau \in R$, *the following inequality holds*

$$0 \le |\tau| - \tau \tanh\left(\frac{\tau}{\xi}\right) \le 0.2785\xi. \tag{3}$$

2.2 | Finite-time performance function

To ensure the tracking error converges to a predefined region in a finite time, the definition of FTPF is introduced as follows

Definition 1 (43). The continuous function $\psi(t)$ is said to be FTPF, if it satisfies the following conditions:

1) $\psi(t) > 0$ and $\dot{\psi}(t) < 0$, which means $\psi(t)$ is a strictly monotonically decreasing positive function;
2) there exists a setting time T_c, such that $\lim_{t \to T_c} \psi(t) = \psi_{T_c}$, and for $\forall t \ge T_c$, $\psi = \psi_{T_c}$, where $\psi_{T_c} > 0$ is a arbitrarily small positive constant.

In this paper, a FTPF is chosen as

$$\psi(t) = \begin{cases} -\tanh\left(\psi_1 + \frac{t}{T_c - t}\right) + \psi_2 + 1, & 0 \le t < T_c \\ \psi_2, & t \ge T_c, \end{cases} \tag{4}$$

where ψ_1, ψ_2, T_c are positive design constants, and $\tanh(\cdot)$ represents the hyperbolic tangent function.

Remark 1. It should be emphasized that $\psi(t)$ has the property of finite-time convergence, while the regular performance function proposed in Reference 34 does not have this property. Moreover, it is clear from (4) that $\psi(t)$ is easier to implement due to the mild initial condition $\psi(0) = -\tanh(\psi_1) + \psi_2 + 1 > 0$ and the independence of system order n.

The objective (2) can be achieved by limiting the tracking error $e_1(t) = x_1(t) - y_d(t)$ to the interval $(-\varsigma_1\psi(t), \varsigma_2\psi(t))$, namely

$$-\varsigma_1\psi(t) < e_1(t) < \varsigma_2\psi(t), \tag{5}$$

where $\varsigma_1 > 0$ and $\varsigma_2 > 0$ are positive design constants, and $\psi(t)$ is a FTPF described as (4).

According to (4) and (5), it is known that $-\varsigma_1\psi(0)$ and $\varsigma_2\psi(0)$ denote the minimum value and the maximum value of the transient undershoot of the tracking error $e_1(t)$, respectively. In addition, T_c represents the time of the tracking error $e_1(t)$ decaying to the steady-state value ψ_2.

Lemma 3 (42). *$\psi(t)$, $\dot{\psi}(t)$ are continuously differentiable and bounded on $[0, +\infty)$, and $\ddot{\psi}(t)$ is continuous and bounded on $[0, +\infty)$.*

2.3 | Stability theory preparation

To introduce the definitions and theorems of stochastic nonlinear systems, consider the following general stochastic system

$$d\boldsymbol{x} = f(\boldsymbol{x})dt + g(\boldsymbol{x})d\omega, \tag{6}$$

where $\boldsymbol{x} \in R^n$ represents the state vector, and ω denotes an r-dimensional independent standard Wiener process. $f(\cdot) : R^n \to R^n$ and $g(\cdot) : R^n \to R^{n\times r}$ are locally Lipschitz functions satisfying $f(\mathbf{0}) = \mathbf{0}$ and $g(\mathbf{0}) = \mathbf{0}$.

Definition 2 (15). For any given $V(\boldsymbol{x}) \in C^2$, LV means the differential operator of V associated with the stochastic system (6), which is defined in the following form

$$LV = \frac{\partial V}{\partial \boldsymbol{x}} f + \frac{1}{2} Tr\left\{ g^{\mathrm{T}} \frac{\partial^2 V}{\partial \boldsymbol{x}^2} g \right\}, \tag{7}$$

where C^2 represents the set of all functions with continuous second-order partial derivative and $Tr\{\cdot\}$ represents the trace of $\cdot$.

Lemma 4 (1). *Consider the stochastic system (6), if there exists a positive definite, radially unbounded, twice continuously differentiable Lyapunov function $V(\boldsymbol{x}) : R^n \to R$, and two positive constants $a > 0$, $b > 0$, such that*

$$LV(\boldsymbol{x}) \leq -aV(\boldsymbol{x}) + b, \tag{8}$$

then, the system (6) has a unique solution almost surely, and the system is bounded in probability.

2.4 | Multi-dimensional Taylor network

In this paper, the unknown nonlinear functions in the controller design process will be treated with MTN. In the previous works,[47–49] the theory related to MTN has been introduced, only the following Lemma is presented.

Lemma 5 (49,50). *Suppose $\varphi(\boldsymbol{s})$ is a continuous nonlinear function defined on a compact set Ω, then for $\forall \varepsilon > 0$, $\varphi(\boldsymbol{s})$ can be approximated by $\boldsymbol{\theta}^T P_{m_n}(\boldsymbol{s})$ as follows*

$$\varphi(\boldsymbol{s}) = \boldsymbol{\theta}^T P_{m_n}(\boldsymbol{s}) + \delta(\boldsymbol{s}), \tag{9}$$

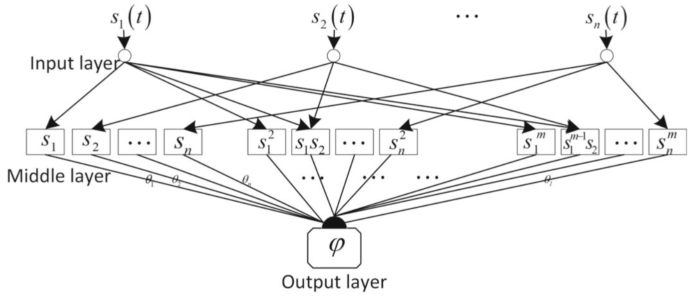

FIGURE 1 The topological structure of MTN.

where $\boldsymbol{s} = [s_1, s_2, \dots, s_n]^{\mathrm{T}}$ *and* $\boldsymbol{\theta} = [\theta_1, \dots, \theta_l]^{\mathrm{T}}$ *represent the input vector and the weight vector of MTN, respectively.* $\delta(\boldsymbol{s})$ *represents the approximation error with* $|\delta(\boldsymbol{s})| \le \varepsilon$. $P_{m_n}(\boldsymbol{s}) = [s_1, \dots, s_n, s_1^2, s_1 s_2, \dots, s_n^2, \dots, s_1^m, s_1^{m-1} s_2, \dots, s_n^m]^{\mathrm{T}}$ *denotes the middle layer of MTN.*

Remark 2. The topological structure of MTN is shown in Figure 1. As a network structure similar to radial basis function neural network (RBFNN),[9] MTN is composed of three layers: input layer, middle layer and output layer. The major difference between the MTN and RBFNN is the way of processing information of the middle layer. Unlike RBFNN, polynomials are adopted instead of radial basis functions in the middle layer of MTN, which can realize the approximation of nonlinear functions with less computation.[13,14]

3 | CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, an event-triggered adaptive MTN controller will be designed for the controlled system (1), which will be addressed in a step-by-step manner. For ease of representation, $f_i(\overline{\boldsymbol{x}}_i)$ and $g_i(\overline{\boldsymbol{x}}_i)$, $i = 1, 2, \dots, n$ will be abbreviated below as f_i and g_i, respectively.

At the start, the following coordinate transformation is defined

$$z_i = x_i - \alpha_{i-1}, i = 2, 3, \dots, n \tag{10}$$

where α_{i-1} represents the virtual control signal to be designed later.

3.1 | Design of control strategy

Step 1: To convert the constrained tracking error $e_1(t)$ into an equivalent unconstrained variable z_1, similar to the methodology of References 42,43, a smooth and strictly increasing transformation function $\gamma(z_1)$ is introduced as follows

$$\gamma(z_1) = \frac{\varsigma_2 e^{z_1} - \varsigma_1 e^{-z_1}}{e^{z_1} + e^{-z_1}}, \tag{11}$$

which satisfies

$$\begin{cases} -\varsigma_1 < \gamma(z_1) < \varsigma_2 \\ \lim\limits_{z_1 \to +\infty} \gamma(z_1) = \varsigma_2, \lim\limits_{z_1 \to -\infty} \gamma(z_1) = -\varsigma_1. \end{cases} \tag{12}$$

In addition, (11) can be transformed into another form as follows

$$\gamma(z_1) = \varsigma_2 - \frac{\varsigma_1 + \varsigma_2}{e^{2z_1} + 1}. \tag{13}$$

Based on (13), we have

$$e^{2z_1} = \frac{\varsigma_1 + \gamma(z_1)}{\varsigma_2 - \gamma(z_1)}. \tag{14}$$

According to (5), (11), and (12), $e_1(t)$ can be expressed as

$$e_1(t) = \psi(t)\gamma(z_1). \tag{15}$$

From (14), we can further obtain $z_1 = \frac{1}{2}\ln\left(\frac{\varsigma_1+\gamma(z_1)}{\varsigma_2-\gamma(z_1)}\right)$, and from (15), we can obtain $\gamma(z_1) = \frac{e_1(t)}{\psi(t)}$. Therefore, the following equation holds

$$z_1 = \frac{1}{2}\ln\left(\frac{\varsigma_1 + \gamma(z_1)}{\varsigma_2 - \gamma(z_1)}\right) = \frac{1}{2}\ln\left(\frac{\varsigma_1 + \frac{e_1(t)}{\psi(t)}}{\varsigma_2 - \frac{e_1(t)}{\psi(t)}}\right). \tag{16}$$

Remark 3. Clearly, the variable z_1 is unconstrained. From (16), we can get that $\ln\left(\frac{\varsigma_1+\frac{e_1(t)}{\psi(t)}}{\varsigma_2-\frac{e_1(t)}{\psi(t)}}\right) = 0$ when $z_1 \to 0$, which implies $e_1(t) = \frac{1}{2}\psi(t)(\varsigma_2 - \varsigma_1)$. Therefore, the tracking error $e_1(t)$ converges to $(-\varsigma_1\psi(t), \varsigma_2\psi(t))$ when $z_1 \to 0$. In addition, according to (15) and (16), since $\psi(t)$ is a strictly monotonically decreasing positive function, $e_1(t)$ can be confined to the following set Δ in a finite time T_c, that is

$$\Delta = \{e_1(t) \in R : |e_1(t)| < \max(\varsigma_1, \varsigma_2)\psi_2, t \geq T_c\}. \tag{17}$$

According to (16), one has

$$\dot{z}_1 = \zeta\left[\dot{e}_1(t) - e_1(t)\frac{\dot{\psi}(t)}{\psi(t)}\right], \tag{18}$$

where $\zeta = \left[\frac{1}{2\psi(t)}\right]\left[\frac{1}{\varsigma_1+\gamma(z_1)} + \frac{1}{\varsigma_2-\gamma(z_1)}\right]$. Obviously, $\zeta > 0$.
From the system model (1) and (18), we have

$$dz_1 = \zeta\left[x_2 + f_1 - \dot{y}_d - e_1(t)\frac{\dot{\psi}(t)}{\psi(t)}\right]dt + \zeta g_1^{\mathrm{T}} d\omega. \tag{19}$$

Consider the first Lyapunov function candidate V_1 as follows

$$V_1 = \frac{1}{4}z_1^4 + \frac{1}{2}\tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_1, \tag{20}$$

where θ_1 denotes the weight vector of MTN and $\hat{\theta}_1$ denotes the estimation of θ_1. $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ denotes the parameter error vector, and $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$ is a constant matrix.
Then, according to Definition 2 and (19), we have

$$LV_1 = z_1^3\zeta\left[x_2 + f_1 - \dot{y}_d - e_1(t)\frac{\dot{\psi}(t)}{\psi(t)}\right] + \frac{3}{2}z_1^2\zeta^2 g_1^{\mathrm{T}} g_1 - \tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\theta}}_1. \tag{21}$$

By using Lemma 1, the following inequality holds

$$\frac{3}{2}z_1^2\zeta^2 g_1^{\mathrm{T}} g_1 \leq \frac{3}{4l_1^2}z_1^4\zeta^4\|g_1\|^4 + \frac{3}{4}l_1^2, \tag{22}$$

where $l_1 > 0$ is a constant.

Substituting (22) into (21), one has

$$LV_1 \le z_1^3\zeta\left[x_2 + f_1 - \dot{y}_d - e_1(t)\frac{\dot{\psi}(t)}{\psi(t)}\right] + \frac{3}{4l_1^2}z_1^4\zeta^4\|g_1\|^4 + \frac{3}{4}l_1^2 - \tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\theta}}_1$$
$$\le z_1^3\zeta x_2 + z_1^3\bar{f}_1 - \frac{3}{4}z_1^4 + \frac{3}{4}l_1^2 - \tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\theta}}_1, \tag{23}$$

where $\bar{f}_1 = \zeta\left[f_1 - \dot{y}_d - e_1(t)\frac{\dot{\psi}(t)}{\psi(t)}\right] + \frac{3}{4}z_1 + \frac{3}{4l_1^2}z_1\zeta^4\|g_1\|^4$.

It is worth noting that $\bar{f}_1$ contains unknown nonlinear functions, which cannot be directly used to design the controller. According to Lemma 5, for any given $\varepsilon_1 > 0$, there exists a $\theta_1^{\mathrm{T}}P_{m1}(Z_1)$, such that

$$\bar{f}_1 = \theta_1^{\mathrm{T}}P_{m1}(Z_1) + \delta_1(Z_1), |\delta_1(Z_1)| \le \varepsilon_1, \tag{24}$$

where $\delta_1(Z_1)$ represents the approximation error, $Z_1 = [z_1]^{\mathrm{T}}$.

Combining (10) and (24), (23) can be reduced to the following form

$$LV_1 \le z_1^3\zeta z_2 + z_1^3\zeta\alpha_1 + z_1^3\theta_1^{\mathrm{T}}P_{m1}(Z_1) + z_1^3\delta_1(Z_1) - \frac{3}{4}z_1^4 + \frac{3}{4}l_1^2 - \tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\theta}}_1. \tag{25}$$

By using Lemma 1 again, we can obtain

$$z_1^3\zeta z_2 \le \frac{3}{4}z_1^4 + \frac{1}{4}\zeta^4 z_2^4, \tag{26}$$

$$z_1^3\delta_1(Z_1) \le |z_1|^3\varepsilon_1 \le \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4. \tag{27}$$

Design the first virtual control signal α_1 as follows

$$\alpha_1 = \frac{1}{\zeta}\left(-k_1z_1 - \hat{\theta}_1^{\mathrm{T}}P_{m1}(Z_1)\right), \tag{28}$$

where $k_1 > \frac{3}{4}$ is a design constant.

The first adaptive law $\dot{\hat{\theta}}_1$ is constructed in the following form

$$\dot{\hat{\theta}}_1 = \Gamma_1 P_{m1}(Z_1)z_1^3 - \Gamma_1\eta_1\hat{\theta}_1, \tag{29}$$

where $\eta_1 > 0$ is a design constant.

By substituting (26)–(29) into (25), the following inequality is easily established

$$LV_1 \le -\left(k_1 - \frac{3}{4}\right)z_1^4 + \frac{1}{4}\zeta^4 z_2^4 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}l_1^2 + \eta_1\tilde{\theta}_1^{\mathrm{T}}\hat{\theta}_1. \tag{30}$$

Step 2: Based on (1) and (10), we have

$$dz_2 = (x_3 + f_2 - L\alpha_1)dt + \left(g_2 - \frac{\partial\alpha_1}{\partial x_1}g_1\right)^{\mathrm{T}} d\omega, \tag{31}$$

where $L\alpha_1 = \frac{\partial\alpha_1}{\partial x_1}(x_2 + f_1) + \sum_{k=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(k)}}y_d^{(k+1)} + \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \frac{1}{2}\frac{\partial^2\alpha_1}{\partial x_1^2}g_1^{\mathrm{T}}g_1$.

Consider the second Lyapunov function candidate as follows

$$V_2 = V_1 + \frac{1}{4}z_2^4 + \frac{1}{2}\tilde{\theta}_2^{\mathrm{T}}\Gamma_2^{-1}\tilde{\theta}_2, \tag{32}$$

where θ_2 denotes the weight vector of MTN and $\hat{\theta}_2$ denotes the estimation of θ_2. $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$ denotes the parameter error vector, and $\Gamma_2 = \Gamma_2^{\mathrm{T}} > 0$ is a constant matrix.

Then, according to Definition 2 and (31), we have

$$LV_2 = LV_1 + z_2^3(x_3 + f_2 - L\alpha_1) + \frac{3}{2}z_2^2\left(g_2 - \frac{\partial\alpha_1}{\partial x_1}g_1\right)^{\mathrm{T}}\left(g_2 - \frac{\partial\alpha_1}{\partial x_1}g_1\right) - \tilde{\theta}_2^{\mathrm{T}}\Gamma_2^{-1}\dot{\hat{\theta}}_2. \tag{33}$$

Similar to step 1, according to Lemma 1, the following inequality holds

$$\frac{3}{2}z_2^2\left(g_2 - \frac{\partial\alpha_1}{\partial x_1}g_1\right)^{\mathrm{T}}\left(g_2 - \frac{\partial\alpha_1}{\partial x_1}g_1\right) \le \frac{3}{4l_2^2}z_2^4\left\|g_2 - \frac{\partial\alpha_1}{\partial x_1}g_1\right\|^4 + \frac{3}{4}l_2^2, \tag{34}$$

where $l_2 > 0$ is a constant.

According to (30) and (34), (33) can be converted into the following form

$$LV_2 \le -\left(k_1 - \frac{3}{4}\right)z_1^4 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}\sum_{j=1}^{2} l_j^2 + \eta_1\tilde{\theta}_1^{\mathrm{T}}\hat{\theta}_1 + z_2^3\left(x_3 + \bar{f}_2\right) - \frac{1}{2}z_2^4 - \tilde{\theta}_2^{\mathrm{T}}\Gamma_2^{-1}\dot{\hat{\theta}}_2, \tag{35}$$

where $\bar{f}_2 = f_2 - L\alpha_1 + \frac{3}{4l_2^2}z_2\left\|g_2 - \frac{\partial\alpha_1}{\partial x_1}g_1\right\|^4 + \frac{1}{2}z_2 + \frac{1}{4}\zeta^4 z_2$.

Similarly, $\bar{f}_2$ cannot be directly used in the design of the controller. According to Lemma 5, $\theta_2^{\mathrm{T}}P_{m2}(\mathbf{Z}_2)$ is used to approximate $\bar{f}_2$. Specifically, for any given $\varepsilon_2 > 0$, the following inequality holds

$$\bar{f}_2 = \theta_2^{\mathrm{T}}P_{m2}(\mathbf{Z}_2) + \delta_2(\mathbf{Z}_2), |\delta_2(\mathbf{Z}_2)| \le \varepsilon_2, \tag{36}$$

where $\delta_2(\mathbf{Z}_2)$ represents the approximation error, $\mathbf{Z}_2 = [z_1, z_2]^{\mathrm{T}}$.

Combining (10) and (35), (36), we have

$$LV_2 \le -\left(k_1 - \frac{3}{4}\right)z_1^4 + \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}\sum_{j=1}^{2} l_j^2 + \eta_1\tilde{\theta}_1^{\mathrm{T}}\hat{\theta}_1 + z_2^3(z_3 + \alpha_2)$$
$$+ z_2^3\theta_2^{\mathrm{T}}P_{m2}(\mathbf{Z}_2) + z_2^3\delta_2(\mathbf{Z}_2) - \frac{1}{2}z_2^4 - \tilde{\theta}_2^{\mathrm{T}}\Gamma_2^{-1}\dot{\hat{\theta}}_2. \tag{37}$$

The second virtual control signal α_2 and the adaptive law $\dot{\hat{\theta}}_2$ are constructed as follows

$$\alpha_2 = -k_2 z_2 - \hat{\theta}_2^{\mathrm{T}}P_{m2}(\mathbf{Z}_2), \tag{38}$$

$$\dot{\hat{\theta}}_2 = \Gamma_2 P_{m2}(\mathbf{Z}_2)z_2^3 - \Gamma_2\eta_2\hat{\theta}_2, \tag{39}$$

where $k_2 > 1$ and $\eta_2 > 0$ are design constants.

By quoting Lemma 1 and substituting (38) and (39) into (37), the following inequality is obtained

$$LV_2 \le -\left(k_1 - \frac{3}{4}\right)z_1^4 - (k_2 - 1)z_2^4 + \frac{1}{4}\sum_{j=1}^{2}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{2} l_j^2 + \sum_{j=1}^{2}\eta_j\tilde{\theta}_j^{\mathrm{T}}\hat{\theta}_j + \frac{1}{4}z_3^4. \tag{40}$$

Step i $(3 \le i \le n-1)$: Similar to step 2, from (1) to (10), we have

$$dz_i = (x_{i+1} + f_i - L\alpha_{i-1})dt + \left(g_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}g_k\right)^{\mathrm{T}} d\omega, \tag{41}$$

where $L\alpha_{i-1} = \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(x_{k+1} + f_k) + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k)}}y_d^{(k+1)} + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_{i-1}}\dot{\hat{\theta}}_{i-1} + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}g_p^{\mathrm{T}}g_q$.

Consider the i-th Lyapunov function candidate as follows

$$V_i = V_{i-1} + \frac{1}{4}z_i^4 + \frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i, \tag{42}$$

where θ_i denotes the weight vector of MTN and $\hat{\theta}_i$ denotes the estimation of θ_i. $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ denotes the parameter error vector, and $\Gamma_i = \Gamma_i^{\mathrm{T}} > 0$ is a constant matrix.

Then, according to Definition 2 and (41), we have

$$\begin{aligned} LV_i = {} & LV_{i-1} + z_i^3(x_{i+1} + f_i - L\alpha_{i-1}) - \tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\theta}}_i \\ & + \frac{3}{2}z_i^2\left(g_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}g_k\right)^{\mathrm{T}}\left(g_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}g_k\right). \end{aligned} \tag{43}$$

Based on Lemma 1, the following inequality holds

$$\frac{3}{2}z_i^2\left(g_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}g_k\right)^{\mathrm{T}}\left(g_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}g_k\right) \le \frac{3}{4l_i^2}z_i^4\left\|g_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}g_k\right\|^4 + \frac{3}{4}l_i^2, \tag{44}$$

where $l_i > 0$ is a constant.

Similar to step 2, substituting (44) into (43), one has

$$\begin{aligned} LV_i \le {} & -\left(k_1 - \frac{3}{4}\right)z_1^4 - \sum_{j=2}^{i-1}(k_j - 1)z_j^4 + \frac{3}{4}\sum_{j=1}^{i}l_j^2 + z_i^3\left(x_{i+1} + \bar{f}_i\right) \\ & - \frac{1}{2}z_i^4 - \tilde{\theta}_i^T\Gamma_i^{-1}\dot{\hat{\theta}}_i + \sum_{j=1}^{i-1}\eta_j\tilde{\theta}_j^{\mathrm{T}}\hat{\theta}_j + \frac{1}{4}\sum_{j=1}^{i-1}\varepsilon_j^4, \end{aligned} \tag{45}$$

where $\bar{f}_i = f_i - L\alpha_{i-1} + \frac{3}{4l_i^2}z_i\left\|g_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}g_k\right\|^4 + \frac{3}{4}z_i$.

Similarly, $\bar{f}_i$ cannot be directly used in the design of the controller. According to Lemma 5, $\theta_i^{\mathrm{T}}P_{mi}(Z_i)$ is used to approximate $\bar{f}_i$. Specifically, for any given $\varepsilon_i > 0$, we have

$$\bar{f}_i = \theta_i^{\mathrm{T}}P_{mi}(Z_i) + \delta_i(Z_i), |\delta_i(Z_i)| \le \varepsilon_i, \tag{46}$$

where $\delta_i(Z_i)$ represents the approximation error, $Z_i = [z_1, \ldots, z_i]^{\mathrm{T}}$.

Then, the construction of the virtual control signal α_i and the adaptive law $\dot{\hat{\theta}}_i$ are constructed as follows

$$\alpha_i = -k_iz_i - \hat{\theta}_i^{\mathrm{T}}P_{mi}(Z_i), \tag{47}$$

$$\dot{\hat{\theta}}_i = \Gamma_iP_{mi}(Z_i)z_i^3 - \Gamma_i\eta_i\hat{\theta}_i, \tag{48}$$

where $k_i > 1$ and $\eta_i > 0$ are design constants.

By substituting (46)–(48) into (45) and quoting Lemma 1, the following inequality is obtained

$$LV_i \le -\left(k_1 - \frac{3}{4}\right)z_1^4 - \sum_{j=2}^{i}(k_j - 1)z_j^4 + \frac{1}{4}\sum_{j=1}^{i}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{i}l_j^2 + \frac{1}{4}z_{i+1}^4 + \sum_{j=1}^{i}\eta_j\tilde{\theta}_j^T\hat{\theta}_j. \tag{49}$$

Step n: Similar to step i, from (1) to (10), we have

$$dz_n = (u + f_n - L\alpha_{n-1})dt + \left(g_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}g_k\right)^{\mathrm{T}}d\omega, \tag{50}$$

where $L\alpha_{n-1} = \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(x_{k+1} + f_k) + \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}}y_d^{(k+1)} + \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_{n-1}}\dot{\hat{\theta}}_{n-1} + \frac{1}{2}\sum_{p,q=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_p\partial x_q}g_p^{\mathrm{T}}g_q$.

Consider the n-th Lyapunov function candidate as follows

$$V_n = V_{n-1} + \frac{1}{4}z_n^4 + \frac{1}{2}\tilde{\theta}_n^{\mathrm{T}}\Gamma_n^{-1}\tilde{\theta}_n, \tag{51}$$

where θ_n denotes the weight vector of MTN and $\hat{\theta}_n$ denotes the estimation of θ_n. $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$ denotes the parameter error vector, and $\Gamma_n = \Gamma_n^{\mathrm{T}} > 0$ is a constant matrix.

Then, according to Definition 2 and (50), we have

$$LV_n = LV_{n-1} + z_n^3(u + f_n - L\alpha_{n-1}) - \tilde{\theta}_n^{\mathrm{T}}\Gamma_n^{-1}\dot{\hat{\theta}}_n + \frac{3}{2}z_n^2\left(g_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}g_k\right)^{\mathrm{T}}\left(g_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}g_k\right). \tag{52}$$

With the aid of Lemma 1, the following inequality holds

$$\frac{3}{2}z_n^2\left(g_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}g_k\right)^{\mathrm{T}}\left(g_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}g_k\right) \le \frac{3}{4l_n^2}z_n^4\left\|g_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}g_k\right\|^4 + \frac{3}{4}l_n^2, \tag{53}$$

where $l_n > 0$ is a constant.

Substituting (53) into (52), one has

$$LV_n \le -\left(k_1 - \frac{3}{4}\right)z_1^4 - \sum_{j=2}^{n-1}(k_j - 1)z_j^4 + \frac{3}{4}\sum_{j=1}^{n} l_j^2 + z_n^3\left(u + \bar{f}_n\right) + \frac{1}{4}z_n^4 - \tilde{\theta}_n^{T}\Gamma_n^{-1}\dot{\hat{\theta}}_n + \sum_{j=1}^{n-1}\eta_j\tilde{\theta}_j^{\mathrm{T}}\hat{\theta}_j + \frac{1}{4}\sum_{j=1}^{n-1}\varepsilon_j^4, \tag{54}$$

where $\bar{f}_n = f_n - L\alpha_{n-1} + \frac{3}{4l_n^2}z_n\left\|g_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}g_k\right\|^4$.

Since $\bar{f}_n$ contains unknown functions, $\bar{f}_n$ cannot be directly used in the design of the controller. According to Lemma 5, for any given $\varepsilon_n > 0$, $\theta_n^{\mathrm{T}}P_{mn}(\boldsymbol{Z}_n)$ is used to approximate $\bar{f}_n$ as follows

$$\bar{f}_n = \theta_n^{\mathrm{T}}P_{mn}(\boldsymbol{Z}_n) + \delta_n(\boldsymbol{Z}_n), |\delta_n(\boldsymbol{Z}_n)| \le \varepsilon_n, \tag{55}$$

where $\delta_n(\boldsymbol{Z}_n)$ represents the approximation error, $\boldsymbol{Z}_n = [z_1, \ldots, z_n]^{\mathrm{T}}$.

Similar to step i, by substituting (55) into (54) and quoting Lemma 1, we can obtain

$$LV_n \le -\left(k_1 - \frac{3}{4}\right)z_1^4 - \sum_{j=2}^{n-1}(k_j - 1)z_j^4 + \frac{1}{4}\sum_{j=1}^{n}\varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{n} l_j^2 + z_n^3 u + z_n^3\theta_n^{\mathrm{T}}P_{mn}(\boldsymbol{Z}_n) + z_n^4 - \tilde{\theta}_n^{\mathrm{T}}\Gamma_n^{-1}\dot{\hat{\theta}}_n + \sum_{j=1}^{n-1}\eta_j\tilde{\theta}_j^{\mathrm{T}}\hat{\theta}_j. \tag{56}$$

Based on the above analysis, the event-triggered control strategy is designed as follows

$$v(t) = -(1+\lambda)\left[\alpha_n \tanh\left(\frac{z_n^3\alpha_n}{\rho}\right) + m' \tanh\left(\frac{z_n^3 m'}{\rho}\right)\right], \tag{57}$$

$$\alpha_n = -k_n z_n - \hat{\theta}_n^{\mathrm{T}}P_{mn}(\boldsymbol{Z}_n), \tag{58}$$

$$\dot{\hat{\theta}}_n = \Gamma_n P_{mn}(\boldsymbol{Z}_n)z_n^3 - \Gamma_n\eta_n\hat{\theta}_n, \tag{59}$$

$$u(t) = v(t_k), \forall t \in [t_k, t_{k+1}), \tag{60}$$

$$t_{k+1} = \inf\{t \in R \| e(t)| \geq \lambda |u(t)| + m\}, e(t) = v(t) - u(t), \tag{61}$$

where $0 < \lambda < 1, m > 0, m' > \frac{m}{1-\lambda}, \eta_n > 0, k_n > 1$ and ρ are positive design constants, $t_k\left(k \in Z^+\right)$ represents the controller update time.

According to (60) and (61), for $\forall t \in [t_k, t_{k+1})$, one has

$$v(t) = [1 + w_1(t)\lambda]u(t) + w_2(t)m, \tag{62}$$

where $w_1(t)$ and $w_2(t)$ are time-varying parameters with $|w_1(t)| \leq 1$ and $|w_2(t)| \leq 1$.

Considering the definition and property of the function $\tanh(\cdot)$, it is easy to know that $\forall \iota \in R, \zeta > 0, -\iota \tanh(\iota/\zeta) \leq 0$. Therefore, based on $\rho > 0$ and $0 < \lambda < 1$, we can conclude $z_n^3 v(t) = (1+\lambda)\left[-\alpha_n z_n^3 \tanh\left(\frac{z_n^3 \alpha_n}{\rho}\right) - m' z_n^3 \tanh\left(\frac{z_n^3 m'}{\rho}\right)\right] \leq 0$. Furthermore, taking $|w_1(t)| \leq 1$ and $|w_2(t)| \leq 1$ into account, the following two inequalities hold

$$\frac{z_n^3 v(t)}{1 + w_1(t)\lambda} \leq \frac{z_n^3 v(t)}{1+\lambda}, \tag{63}$$

$$-\frac{z_n^3 w_2(t) m}{1 + w_1(t)\lambda} \leq \left|\frac{z_n^3 m}{1-\lambda}\right|. \tag{64}$$

According to $m' > \frac{m}{1-\lambda}$, we have $-\left|m' z_n^3\right| + \left|\frac{z_n^3 m}{1-\lambda}\right| < 0$. Then, based on (57)–(64) and Lemma 2, one has

$$\begin{aligned}
z_n^3 u &= \frac{z_n^3 v(t)}{1 + w_1(t)\lambda} - \frac{z_n^3 w_2(t) m}{1 + w_1(t)\lambda} \\
&\leq -\alpha_n z_n^3 \tanh\left(\frac{z_n^3 \alpha_n}{\rho}\right) - m' z_n^3 \tanh\left(\frac{z_n^3 m'}{\rho}\right) + \left|\frac{z_n^3 m}{1-\lambda}\right| \\
&\leq \left|\alpha_n z_n^3\right| - \alpha_n z_n^3 \tanh\left(\frac{z_n^3 \alpha_n}{\rho}\right) + \left|m' z_n^3\right| - m' z_n^3 \tanh\left(\frac{z_n^3 m'}{\rho}\right) + \alpha_n z_n^3 - \left|m' z_n^3\right| + \left|\frac{z_n^3 m}{1-\lambda}\right| \\
&\leq 0.2785\rho + 0.2785\rho + \alpha_n z_n^3 \\
&\leq 0.557\rho + \alpha_n z_n^3.
\end{aligned} \tag{65}$$

Substituting (58), (59) and (65) into (56), the following inequality holds

$$LV_n \leq -\left(k_1 - \frac{3}{4}\right) z_1^4 - \sum_{j=2}^{n} \left(k_j - 1\right) z_j^4 + \frac{1}{4}\sum_{j=1}^{n} \varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{n} l_j^2 + 0.557\rho + \sum_{j=1}^{n} \eta_j \tilde{\theta}_j^{\mathrm{T}} \hat{\theta}_j. \tag{66}$$

Then, by quoting Lemma 1, we have

$$\sum_{j=1}^{n} \eta_j \tilde{\theta}_j^{\mathrm{T}} \hat{\theta}_j \leq -\sum_{j=1}^{n} \frac{\eta_j}{2\lambda_{\max}\left(\Gamma_j^{-1}\right)} \tilde{\theta}_j^{\mathrm{T}} \Gamma_j^{-1} \tilde{\theta}_j + \frac{1}{2}\sum_{j=1}^{n} \eta_j \left\|\theta_j\right\|^2. \tag{67}$$

Substituting (67) into (66), we can obtain

$$LV_n \leq -\sum_{j=1}^{n} c_j z_j^4 - \frac{1}{2}\sum_{j=1}^{n} \overline{\eta}_j \tilde{\theta}_j^{\mathrm{T}} \Gamma_j^{-1} \tilde{\theta}_j + \frac{1}{4}\sum_{j=1}^{n} \varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{n} l_j^2 + \frac{1}{2}\sum_{j=1}^{n} \eta_j \left\|\theta_j\right\|^2 + 0.557\rho, \tag{68}$$

where $c_1 = k_1 - \frac{3}{4}, c_j = k_j - 1 (j = 2, 3, \ldots, n), \overline{\eta}_j = \min\{(\eta_j/(\lambda_{\max}(\Gamma_j^{-1})))|j = 1, 2, \ldots, n\}$.

3.2 Stability analysis

The stability analysis of the controlled system (1) is presented by the following theorem.

Theorem 1. *Under Assumption 1, considering the controlled system (1) with the predetermined performance (5), for any initial values $e_1(0)$ satisfying (5), there exists a control strategy composed by the virtual control signals (28), (38), (47) and (58), the adaptive laws (29), (39), (48) and (59), the adaptive controller (57) with the event-triggered conditions (60) and (61), such that:*

(1) *all the closed-loop signals are bounded on $[0, +\infty)$ in probability.*
(2) *the tracking error e_1 converges to a small residual set of zero with performance constraint (5).*
(3) *the Zeno behavior is avoided successfully.*

Proof. In the first place, we prove that all signals of the closed-loop system (1) are bounded in probability.
The whole Lyapunov function candidate V is constructed as

$$V = \sum_{k=1}^{n} \frac{1}{4} z_k^4 + \sum_{k=1}^{n} \frac{1}{2} \tilde{\theta}_k^{\mathrm{T}} \Gamma_k^{-1} \tilde{\theta}_k. \tag{69}$$

From (68) and (69), we have

$$LV \leq -a_0 V + b_0. \tag{70}$$

where $a_0 = \min\left\{4c_j, \overline{\eta}_j | j = 1, 2, \ldots, n\right\}$, and $b_0 = \frac{1}{4}\sum_{j=1}^{n} \varepsilon_j^4 + \frac{3}{4}\sum_{j=1}^{n} l_j^2 + 0.557\xi + \frac{1}{2}\sum_{j=1}^{n} \eta_j \left\|\theta_j\right\|^2$.
According to Lemma 4 and (70), we can conclude that V is bounded in probability. Then, taking expectations on both sides of (70) and multiplying it by $e^{a_0 t}$, one has $d\left(e^{a_0 t}E(V)\right) \leq b_0 e^{a_0 t}$. Furthermore, integrating it on $[0, t]$ as follows

$$E[V] \leq E(V(0))e^{-a_0 t} + \frac{b_0}{a_0} \leq E(V(0)) + \frac{b_0}{a_0}. \tag{71}$$

From (69) and (71), the following two inequalities hold

$$E[|z_k|] \leq \sqrt[4]{E(V(0)) + \frac{b_0}{a_0}}, \tag{72}$$

$$E\left[\left|\sum_{k=1}^{n} \frac{1}{2} \tilde{\theta}_k^{\mathrm{T}} \Gamma_k^{-1} \tilde{\theta}_k\right|\right] \leq E(V(0)) + \frac{b_0}{a_0}. \tag{73}$$

Based on (72) and (73), we can conclude that z_k and $\|\tilde{\theta}_k\|$ are bounded in probability. It follows from (28), (38), (47), (57), (58) and (62) that α_i, u, and v are bounded in probability. Since $x_i = z_i + \alpha_{i-1}$, we can conclude that x_i is bounded in probability. The above analysis proves that all signals of the closed-loop system (1) are bounded in probability.

In the second place, we prove that the tracking error converges to a small residual set of zero with performance constraint (5).

From (70), we naturally obtain that

$$LV_1 \leq -a_0 V_1 + b_0. \tag{74}$$

Then, multiplying both sides of (74) by $e^{a_0 t}$ and integrating it on $[0, t]$ as follows

$$V_1(t) \leq a_0^* + \left[V_1(0) - a_0^*\right] e^{-a_0 t}, \tag{75}$$

where $a_0^* = b_0 / a_0$.

According to (75) and (20), we can obtain $\lim_{t\to\infty}|z_1| \le \sqrt[4]{4a_0^*}$. By choosing appropriate design parameters c_j, η_j, ε_j, l_j and matrix Γ_j, z_1 can converge to a small residual set of zero. Based on the error transformation (16), we can easily conclude that the tracking error e_1 can also converge to a small residual set of zero. In practical applications, the design parameters should be chosen appropriately to meet specific requirements.

At last, we need to prove that the proposed controller can avoid the Zeno behavior. In other words, there exists a positive constant t^*, for $\forall k \in Z^+$, such that $\{t_{k+1} - t_k\} \ge t^*$, where t^* denotes the lower bound of the minimum inter-execution interval.

From $e(t) = v(t) - u(t)$, $\forall t \in [t_k, t_{k+1})$, we have

$$\frac{d|e(t)|}{dt} = \frac{d}{dt}(e \times e)^{\frac{1}{2}} = \text{sign}\,(e)\dot{e} \le |\dot{v}|. \tag{76}$$

Based on (57) and (58), we have the conclusion that since all terms contained in α_n are bounded and have continuous derivatives, there exists $\dot{v}$ which is bounded. In other words, there must exist a constant $M > 0$, such that $|\dot{v}| \le M$. From $e(t_k) = 0$ and $\lim_{t\to t_{k+1}} e(t) = \lambda|u(t)| + m > m$, we know that the lower bound of the inter-execution interval t^* must satisfy $t^* \ge m/M > 0$, so Zeno behavior cannot occur.

In summary, the proof of Theorem 1 is complete. ■

Remark 4. Based on the above analysis, it can be seen that our control strategy has significant advantages. In practical engineering, in order to obtain the desired tracking accuracy and convergence time, the user must repeatedly adjust the parameters to achieve the goal, which is a time-consuming and laborious process. However, in this paper, by introducing FTPF, tracking accuracy and convergence time are considered, and they can be predetermined by adjusting the parameters of FTPF.

4 | SIMULATION EXPERIMENT

In this section, a numerical example and a practical example are used to illustrate the effectiveness of the control strategy proposed in this paper.

Example 1. To verify the effectiveness of the proposed control strategy, the following second-order stochastic nonlinear system is considered:

$$\begin{cases} dx_1 = (x_2 + 0.2x_1\sin(x_1))dt + 0.1x_1 d\omega \\ dx_2 = \left(u - x_1\cos\left(x_2^2\right)\right)dt + 0.1x_2 d\omega \\ y = x_1 \end{cases} \tag{77}$$

with the initial state satisfies $x_1(0) = x_2(0) = 0$, the reference trajectory is selected as $y_d = \sin(t)$.

In the simulation, the control structure is chosen in the same way as Theorem 1. The parameters of FTPF $\psi(t)$ are taken as $\psi_1 = 0.8$, $\psi_2 = 0.03$, $T_c = 5$. The parameters of control strategy are designed as $\varsigma_1 = 8.1$, $\varsigma_2 = 8.1$, $\lambda = 0.01$, $\rho = 0.001$, $m' = 3$. The parameters of MTN are selected in two cases, (i) case 1: $k_1 = 9$, $k_2 = 100$, $\eta_1 = 12$, $\eta_2 = 20$; (ii) case 2: $k_1 = 1$, $k_2 = 45$, $\eta_1 = 11$, $\eta_2 = 10$. The simulation results are shown in Figures 2–6.

Figure 2 illustrates that the system output y can track the desired reference signal y_d closely under two cases, which indicates that the choice of design parameters may affect the initial tracking performance, but not affect the final tracking performance. The responses of tracking error $e_1(t)$ in two cases are shown in Figure 3, it can be seen that the tracking error can converge to the prescribed performance constraint. Figure 4 shows the boundedness of u and v in case 2. Figure 5 shows the response of the state variable x_2 in case 2. Figure 6 represents the time between two adjacent events, which indicates that Zeno behavior does not occur.

Example 2. In order to verify the applicability of the proposed control strategy, a class of single link robot arm dynamics system is considered. According to the work of Reference 51, its system can be represented as

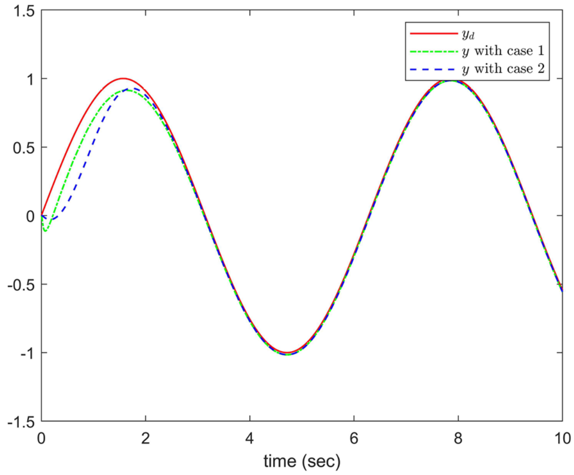

FIGURE 2 The responses of y_d and y of Example 1 in two cases.

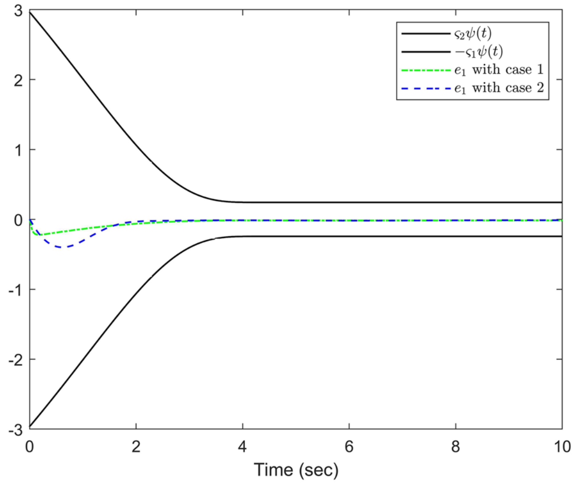

FIGURE 3 The responses of e_1 with performance constraint of Example 1 in two cases.

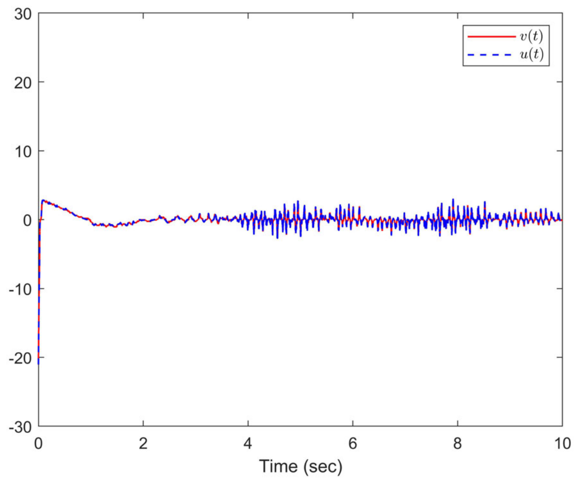

FIGURE 4 The responses of $v(t)$ and $u(t)$ of Example 1 in case 2.

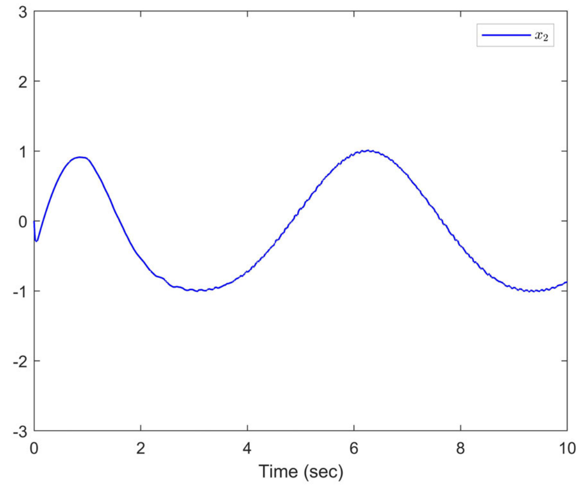

FIGURE 5 The response of state variable x_2 of Example 1 in case 2.

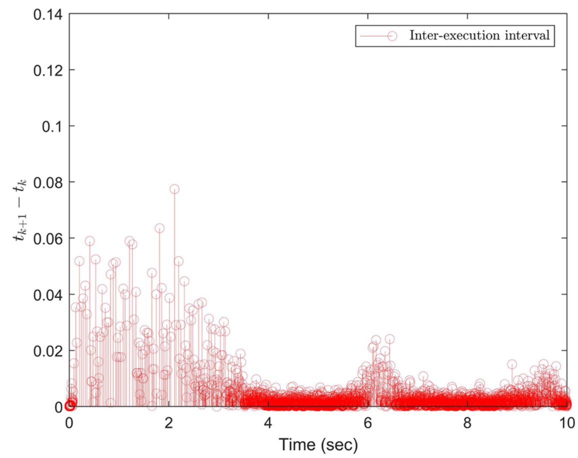

FIGURE 6 The response of inter-execution interval of Example 1 in case 2.

a second-order stochastic nonlinear system of the following form

$$\begin{cases} dx_1 = x_2 dt \\ dx_2 = (u - 5\sin(x_1))dt + 0.5x_2 \sin(x_1)d\omega \\ y = x_1 \end{cases} \tag{78}$$

where the initial condition is $x_1(0) = x_2(0) = 0$.

In the simulation, the reference trajectory is selected as $y_d = \sin(t)$. The control structure is chosen in the same way as Theorem 1. The tracking error e_1 satisfies the predefined performance constraint: $-2.7\psi(t) < e_1(t) < 2.7\psi(t)$, and the design parameters take the following values: $\psi_1 = 0.8$, $\psi_2 = 0.03$, $T_c = 5$, $\lambda = 0.01$, $\rho = 0.001$, $m' = 3$, $k_1 = 1$, $k_2 = 45$. The simulation results of the system (78) are shown in Figures 7–11.

Figure 7 displays the responses of the system output y and the reference signal y_d, which shows that the satisfying tracking performance can be achieved. Figure 8 indicates that the tracking error e_1 converges to a small residual set of zero with finite-time performance constraint. Figures 9 and 10 show the responses of $v(t)$, $u(t)$ and x_2, respectively. Figure 11 displays the response of inter-execution interval, which indicates that Zeno behavior has been avoided successfully. The simulation results further verify the feasibility of the control strategy proposed in this paper.

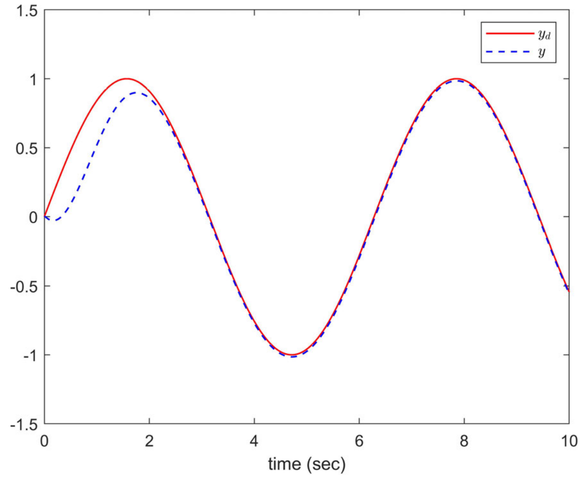

FIGURE 7 The responses of y_d and y of Example 2.

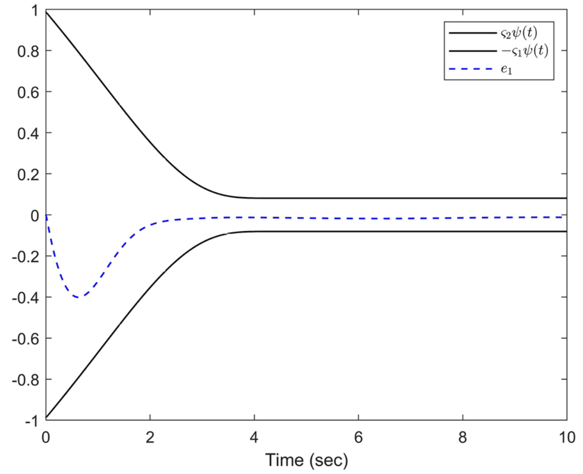

FIGURE 8 The response of e_1 with performance constraint of Example 2.

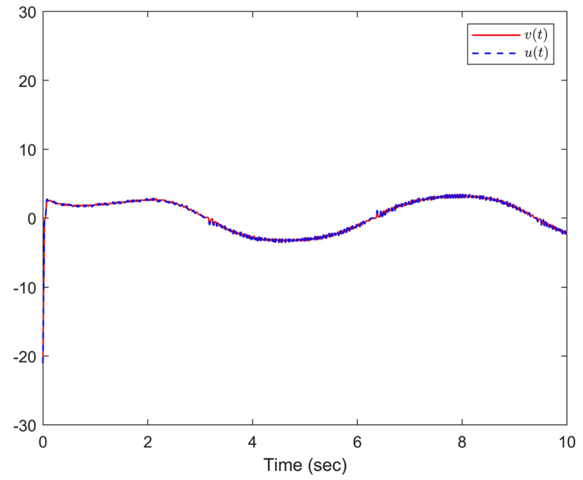

FIGURE 9 The responses of $v(t)$ and $u(t)$ of Example 2.

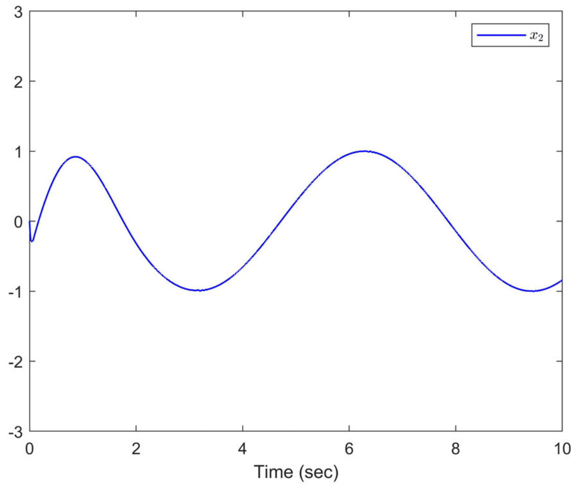

FIGURE 10 The response of state variable x_2 of Example 2.

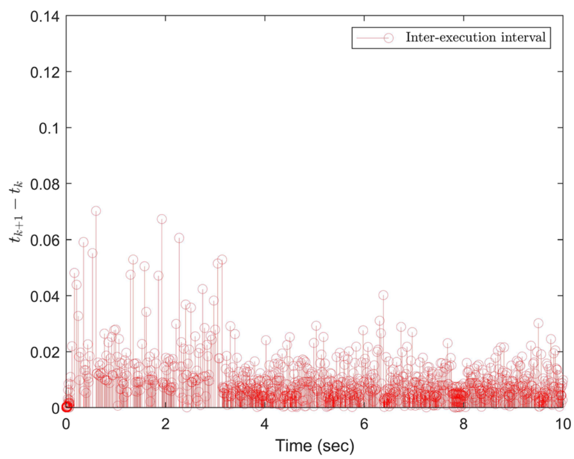

FIGURE 11 The response of inter-execution interval of Example 2.

Remark 5. The aforementioned stability analysis and simulations show that the control performance of system can be influenced by the design parameters chosen. The tracking performance and convergence speed can be enhanced by selecting the optimal design parameters. Therefore, in order to more successfully meet the specific control objectives, the real engineering system should be carefully adjusted to select the most appropriate parameters.

5 | CONCLUSION

In this paper, the problem of predetermined finite-time performance control has been investigated for strict-feedback stochastic nonlinear systems, in which an easy-to-implement FTPF has been introduced to describe the predefined tracking performance. The original constrained tracking error has been transformed into an equivalent unconstrained variable by means of a transformation function. Moreover, an event-triggered adaptive tracking control strategy has been proposed with the aid of adaptive backstepping method and MTN. Based on the above technologies, this control strategy has guaranteed that all signals of the closed-loop system were bounded in probability. It also has achieved predetermined tracking performance in a finite time and saved communication resources while avoiding Zeno behavior. Finally, simulation results have shown the effectiveness of the proposed control strategy.

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